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PHYSICOMECHANICAL PARAMETERS OF SHELLS OF BODIES OF REVOLUTION THAT ENSURE THE FORMATION OF A RUNNING WAVE ON THEIR SURFACE IN VISCOUS-FLUID FLOW

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The influence of the physicomechanical properties of large-scale shells on their vibration in a fluid flow is investigated. The ranges of the elastic and shear moduli and the Poisson coefficients of the anisotropic material of the shell, for which a running wave is formed on its surface, are determined.

Keywords: *hydroelasticity problem, shell from anisotropic composite material, running wave.*

Introduction. A potential possibility of reducing substantially the hydrodynamic drag of a body if a running wave on the shell's surface is used as a means for controlling the fluid flow has been shown in [1–5]. Conditions under which a running wave can be sustained in a shell of finite length have been partially studied in [6, 7] without allowance for the flow effects. The physicomechanical properties of shells that ensure the formation of a running wave in the case of viscous-fluid flow past them are the focus of the present work.

Formulation of the Problem and Method of Its Solution. Consideration of the behavior of the shell in fluid flow calls for solution of the conjugate problems — those of hydrodynamics and elastic deformation — as a coupled hydroelasticity problem that involves the equation of motion of the elastic shell

$$M\frac{\partial^2 \boldsymbol{\delta}}{\partial t^2} + R\frac{\partial \boldsymbol{\delta}}{\partial t} + K\boldsymbol{\delta} = \mathbf{F}\,(x, t) \tag{1}$$

and the Navier–Stokes equations

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\Delta\mathbf{u}) = -\nabla p + \frac{1}{\mathrm{Re}}\Delta\mathbf{u}\,, \quad \mathrm{div}\,\mathbf{u} = 0\,. \tag{2}$$

The cooperative motion of the flow and the shell is determined by the system of differential equations (1)–(2), which is integrated in the (7 × 3) rectangular region of the chord of the body in flow (its length is equal to unity in normalized variables) with the initial $\mathbf{u}(x, r, 0) = \mathbf{U}_\infty$ and boundary $\mathbf{u}(x, r, t) = \mathbf{U}_\infty$ conditions at the left- and right-hand boundaries of the computational domain, the condition $\partial\mathbf{u}(x, r, t)/\partial t = 0$ at its right-hand boundary, and the conditions $u_n(x, r_0, t) = \partial\delta_n/\partial t$ and $u_s(x, r_0, t) = \partial\delta_s/\partial t$ on the shell's surface.

The displacements of the shell δ_n and δ_s are determined by the equation of motion (1), whose right-hand side contains the pressure p computed from Eq. (2). The conjugate problem is solved for the Reynolds numbers Re = 10^4 and 10^5.

The procedure of solution of the conjugate difference problem is based on the simultaneous use of a Kompozit-2005 software package and a program of integration of Navier–Stokes equations on the basis of the "large-particle" method [2, 8, 9]. In the final analysis, integration of Eqs. (1)–(2) with the corresponding initial and boundary conditions is reduced to solution of the systems of differential equations at the nodes of a computational grid by performing time steps successively. At each step, the equations of motion of the shell are first integrated with respect to

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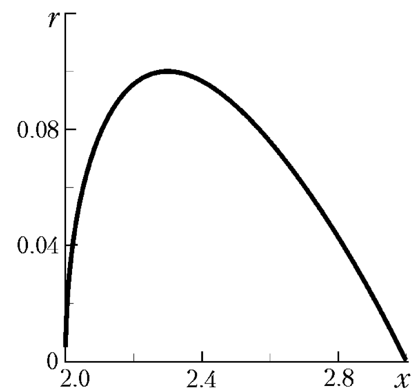


Fig. 1. Meridian of the shell. r, m.

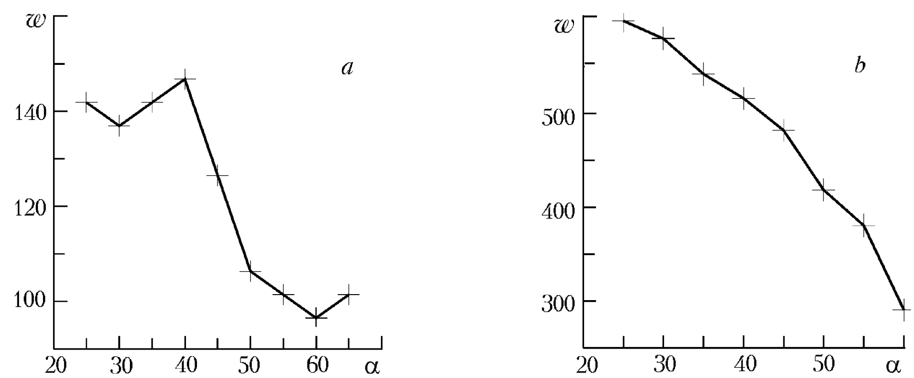


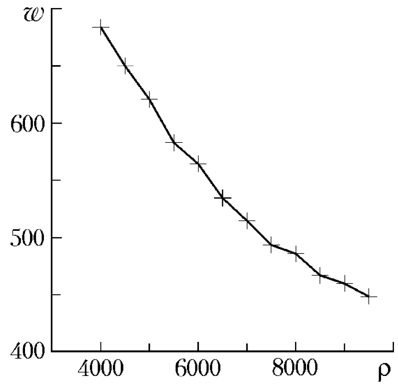
Fig. 2. Phase velocity of the running wave vs. reinforcement angle: a) for meridian displacements, b) for normal displacements. w, m/sec; α, deg.

time using an implicit difference scheme. The resulting displacements determining the running shape of the boundary and the velocity of motion of the flow boundary are transferred to the program which integrates the Navier–Stokes equations and in which pressures are computed for the running time step. Then the calculated pressure is used to obtain displacements at the following time step.

Investigation Results. According to the procedure (described in [6, 7]) of calculation of wave processes in shells of revolution, we study the influence of the model's parameters on the phase velocity of the running wave in a shell whose meridian is presented in Fig. 1 for a prescribed pressure at each point of the meridian and a disturbing force prescribed on the leading edge in a circle. The shell has the following geometric parameters: chord length x_f = 1.0221 and thickness h = 0.001 m. Variation of the parameters of the basic model is carried out with its successive correction to obtain a variant satisfying the final objective of the investigations, i.e., determination of the physicomechanical properties of the shell for which a running wave is formed by the action of the driving force in the case of viscous-fluid flow past the shell. We model an anisotropic material with the following parameters: elastic moduli E_1 = 2900 MPa and $E_2 = E_3$ = 750 MPa, Poisson coefficients ν_1 = 0.2 and $\nu_2 = \nu_3$ = 0.1, shear moduli $G_1 = G_2 = G_3 = 2.5 \cdot 10^8$ Pa, and density ρ = 8000 kg/m^3. The shell is fixed on the leading edge along the axial and circular displacements.

The phase velocity of the running wave as a function of the reinforcement angle α is plotted in Fig. 2a for meridian displacements and in Fig. 2b for normal ones. It is seen that the phase velocity w diminishes with increase in the reinforcement angle; the flexural wave has its maximum at about 40^o but the flexural-wave front lags behind the longitudinal-wave front.

Figure 3 plots the phase velocity of normal displacements of the running wave as a function of the density of the shell material in reinforcement at an angle of 45^o. It is clear from the figure that the phase velocity diminishes in inverse proportion to the root of the density as the density increases. The flexural wave decays on 20% of the meridian.



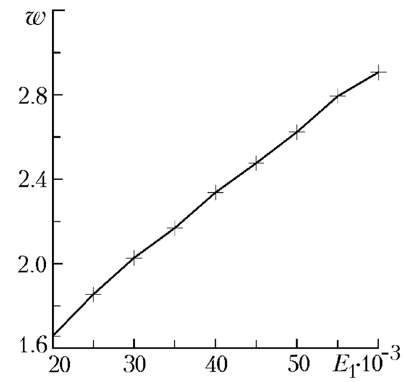


Fig. 3. Phase velocity of the running wave vs. density of the shell material. w, m/sec; ρ, kg/m^3.

Fig. 4. Phase velocity of the running wave vs. elastic modulus of the shell material. w, m/sec; E_1, Pa.

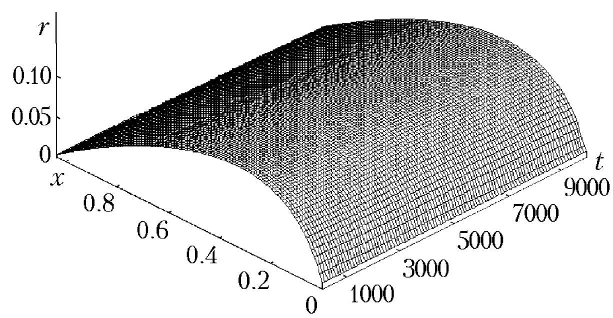


Fig. 5. Shape of the shell in viscous-fluid flow as a function of x and t (time scale on the plot increased 10^4 times) for Reynolds number Re = 10^4 without a disturbing force.

Also, we investigate the change in the phase velocity of the running wave with variation of the parameters of the material. For the phase velocity to have the order of 1–10 m/sec, which is related to the velocity of fluid flow past the shell, the material's parameters are accordingly changed and are as follows: elastic moduli E_1 = 29,000 Pa and $E_2 = E_3$ = 75,000 Pa, Poisson coefficients $\nu_1 = 0.2$ and $\nu_2 = \nu_3 = 0$, shear moduli $G_1 = G_2 = G_3 = 0.0025$ Pa, and density ρ = 8000 kg/m^3.

The investigations have shown that the phase velocity of the running wave is weakly dependent on the Poisson coefficient ν_1 and the elastic moduli E_2 and E_3. With increase in E_1 this velocity increases in proportion to the root of the elastic modulus (Fig. 4). In all the cases considered in variation the running waves in normal displacements decay on 20% of the meridian.

In investigating the influence of the shear moduli on the running-wave velocity, we use a shell with the following parameters of the material: elastic moduli E_1 = 20,000 Pa and $E_2 = E_3$ = 100 Pa, Poisson coefficients ν_1 = 0.1 and $\nu_2 = \nu_3 = 0$, and density ρ = 8000 kg/m^3. The shell is reinforced at an angle of 45°. It is observed that the phase velocity of the wave remains constant with variation of the shear moduli. The running waves in normal displacements do not decay.

On the basis of the preliminary investigations, we select, for solution of the conjugate problem in which the appearance of a running wave with an amplitude of the order of the boundary-layer thickness can be expected, the following physicomechanical parameters of the shell material: thickness h = 0.015 m, elastic moduli E_1 = 20,000 Pa and $E_2 = E_3$ = 100 Pa, Poisson coefficients ν_1 = 0.1 and $\nu_2 = \nu_3 = 0$, shear moduli $G_1 = G_2 = G_3$ = 10 Pa, and density ρ = 8000 kg/m^3. The shell is reinforced at an angle of 45°. The frequency of the disturbing force is ω = 100 Hz and the amplitude is A = 1000 Pa.

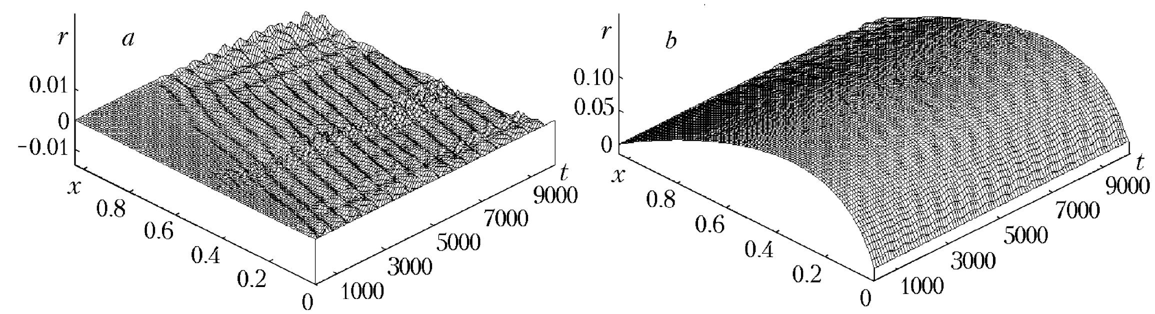


Fig. 6. Radial displacement of the shell (a) and its shape (b) as functions of x and t in the case of viscous-fluid flow past it in the presence of a disturbing force applied along to normal near the forepart.

Figure 5 gives the shape of the shell in viscous-fluid flow as a function of x and t for Reynolds number $\mathrm{Re} = 10^4$ in the absence of a disturbing force. Figure 6 gives the radial displacements and shape of the shell in flow of the same fluid in the presence of a disturbing force applied along the normal near the body's forepart. The time scale is increased 10^4 times in these figures. It is seen that a running wave not decaying in normal displacements is formed. The average amplitudes of normal and radial displacements are similar and equal to ~0.006. Elimination of a certain irregularity in the behavior of the running wave calls for shells with a stiffness variable with length. An increase of an order of magnitude in the Reynolds number preserves qualitatively the same vibrational pattern of the shell, but the amplitudes of normal and radial displacements diminish to ~0.002. We emphasize that the amplitude of the running wave is an order of magnitude smaller when the pressure on the shell is constant (i.e., without viscous-fluid flow past it).

Conclusions. It has been demonstrated that there is a possibility of creating an anisotropic composite material with parameters enabling one to form a running wave on the surface of the shell in viscous-fluid flow. It has been established that the phase velocity of the running wave is substantially influenced by the reinforcement angle, the density of the shell material, and the elastic modulus E_1. The amplitude of vibrations is dependent on the amplitude of the disturbing force, the shell thickness, and the Reynolds number.

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NOTATION

A, amplitude of the disturbing force; E_1, E_2, and E_3, meridian, normal and circular elastic moduli; $\mathbf{F}(x, t)$, disturbing force; G_1, G_2, and G_3, shear moduli; h, shell thickness; K, shell stiffness matrix; M, shell mass matrix; p, pressure; R, shell damping matrix; r, radial coordinate; Re, Reynolds number; r_0, shell radius; t, time; $\mathbf{u}$, flow-velocity vector; $u_n(x, r_0, t)$, normal component of the velocity vector; $u_s(x, r_0, t)$, tangential component of the velocity vector; $\mathbf{U}_\infty$, vector of the incident-flow velocity; w, phase velocity of the elastic wave; x, axis of symmetry; x_f, length of the shell's chord; α, reinforcement angle of the shell; δ, shell displacement vector; δ_n, normal displacement of the shell; δ_s, meridian displacement of the shell; ν_1, ν_2, and ν_3, Poisson coefficients; ρ, material's density; ω, frequency of the disturbing force. Subscripts: 0, initial; f, finite; n, normal; s, tangent to the shell's meridian; ∞, infinity.

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